It is all About Dissimilarity: Party System Characteristics and Their Proper Measurement

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Abstract

In this article we propose a new conceptualization of the crucial party system characteristics: disproportionality, electoral volatility, territorial heterogeneity and inter-election incongruence. We argue that these characteristics can be studied as dissimilarities between vectors of votes or seats. We present different specifications of vectors in order to address various research questions important for students of parties and party systems. Subsequently, developing the analyses of Monroe (1994) and Taagepera and Grofman (2003), we present nine measures of vectors' dissimilarity: index of dissimilarity, Gallagher's least squares measure and its transformations, cosine measure, Gini coefficient, Kullback-Leibler divergence (relative entropy), weighted variance and weighted standard deviation of ratios. We discuss their utility in empirical studies of main party system characteristics, using several dimensions of comparison, based on the formal postulates. We also add two new postulates concerning measure's decomposability: horizontal (sum-type) and vertical (variance-type).

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Introduction

There are six main features of party systems describing their structure and stability which attract the attention of party scholars: fragmentation, polarization, disproportionality, volatility, territorial heterogeneity, and inter-election incongruence. Various indicators, with different formal properties, have been used to measure these features and compare them across time and across countries. In this article, we propose an integrated framework which will help to study, develop, and use in practice the measures of disproportionality, volatility, territorial heterogeneity, and inter-election incongruence, which are based on the electoral data only. We do not take into consideration the polarization of party system which is studied primarily with the use of survey data or party manifestos. We also omit the fragmentation of party system and focus on four other features which are interlinked by a formal kinship. The measurement of fragmentation is based on simple descriptive statistics of a single votes' distribution (Rae, 1967; Laakso, Taagepera, 1979; Golosov, 2012), while the remaining four features of party systems can be operationalized as a degree of dissimilarity between two vectors (distributions of votes or seats). In this article we compare various measures of dissimilarity describing their main statistical properties and discussing their usability in the empirical research on party systems. Our main idea is to present the most frequently used measures within a generalized methodological framework.

In this article, we begin with the review of the above-mentioned party system characteristics, demonstrating that they can be studied as dissimilarities between pairs of vectors containing the electoral results. We speculate about the possible usage of dissimilarities between various vectors in political research and attempt to address some conceptual problems identified in the existing literature. Subsequently, we present nine major measures of dissimilarity, which could be employed to the empirical analyses of electoral data. They share some properties (we define the postulates they all satisfy) and differ on certain other dimensions (we define the postulates which allow to make comparisons between nine studied measures and discuss their practical significance). The article ends with the summary of our assessment.

Party system characteristics as dissimilarities

The similarity between measures of disproportionality, volatility, and split-ticket voting were already noticed by Taagepera and Grofman (2003). These authors reviewed also the properties of various indices used to measure these concepts. We argue that territorial heterogeneity should be added to that list. Moreover, the concept of split-ticket voting should be replaced by inter-election incongruence, which is more general and refers directly to the systemic, not individual level[1].

Let us remind that the ***level of party system proportionality*** is usually determined by the comparison of the votes and seats distributions from the same elections. Theoretically, the extremely proportional is the election in which the distribution of seats among parties is identical to the distribution of votes casted for each of them. The deviations from the ideal situation of equal representation are mainly the effect of the electoral law. For that reason, unsurprisingly, the measurement of disproportionality of party system is crucial in the comparative research of electoral rules and in practical studies focused on the selection of the optimal rules of

[1] Vote-splitting is a concept frequently used in electoral studies. However, it is conceptualized at an individual level and, thus, the empirical studies of vote-splitting generally require the individual-level data, i.e. individual voter's preferences in the elections taking place simultaneously or in a single election in which a voter can cast more than one vote (as in the case of German Bundestag elections). One could attempt to analyze the aggregated vote-splitting in a simultaneous elections with the use of electoral results by determining the level of similarity between the election A and election B. It could be easily interpreted as an analogy to the aggregated electoral volatility. The two following elections of the same representative body would be analogous to two different elections taking part simultaneously (e.g. elections of lower and upper chamber of the parliament). The condition of simultaneity would allow to assume that the same electorate participated in a particular pair of elections. However, one should compute the dissimilarity indices at the lowest possible level of territorial data aggregation (municipalities, electoral wards) in order to minimize the "netto effect". The third index of inter-election dissimilarity (DIS3) between proposed by Schakel (2013) could be treated as an aggregate vote-splitting index, but it does not necessarily fulfill the elections simultaneity criterion and a region, for which Schakel computes dissimilarity indices, is a relatively large territorial unit.

proportional representation systems. Van Puyenbroeck (2008) sketched an important distinction between the two approaches to the problem of disproportionality which are based on different theoretical assumptions. He argues that the original concept of disproportionality stems from the idea of equal votes and its measurement should be focused on the equality of seat/vote ratios among individual voters in proportional representation systems. However, in the comparative political research disproportionality is used as one of the characteristics of party systems, including majoritarian. This approach stems from the idea of "fair elections" focusing rather on equal ("equally treated") parties than equal votes (Grilli di Cortona et al, 1999: ch. 6). Students of party systems focus mainly on the extent to which the distribution of votes collected by parties is similar to the distribution of seats in the elected assembly. As we will demonstrate further, the distinction between these two approaches towards disproportionality, although rarely discussed in practice, plays a relatively important role in the selection of proper measures. Although we share van Puyenbroeck's (2008) terminological reservations, in this article we do not follow his suggestion to use the term "deviation from proportionality" to describe the latter approach. In our opinion, the context (analyses of party system features, not comparison of PR systems) clarifies the meaning of disproportionality sufficiently.

Among numerous indices of disproportionality, the first was the distortion index proposed by Loosemore and Hanby (1971) and equivalent to the measure of income inequality which was used years before by the economists (Duncan, Duncan, 1955). Recently, the most frequently used measure of disproportionality and malapportionment is the least square index proposed by Gallagher (1991). The literature enumerates even more measures; some of them are designed specifically for a particular version of a proportional representation system (Karpov, 2008; Chessa, Fragnelli, 2011; Koppel, Diskin, 2009).

The ***temporal stability of the party system*** depends primarily on the shifts of support for parties taking part in the following elections and it is described by the survey indicators of electoral volatility (measured on the individual level and based on the declarations of voters) or by the aggregated indices of electoral volatility which are based on the comparison of the results of two subsequent elections. Commonly used is the electoral volatility index, proposed by Pedersen (1979), formally equivalent to Loosemore-Hanby index of disproportionality and index of income inequality. While computing the electoral volatility index, one should take into account the differences in vote shares (sometimes – seat shares) gained by particular parties. The higher the index, the larger is the dissimilarity between two following representative bodies. The growing electoral volatility is regarded as a sign of a dynamic party system change (Bartolini, Mair, 1990; Dassoneville, Hooghe, 2011).

The ***internal coherence of party system***, subject to the formulation, could be described either by the ***territorial heterogeneity*** of party support, or by the ***dissimilarity between different levels (layers) of party systems*** (local, regional, sometimes also European). Both of these features are known from the literature on the nationalization of party systems. However, the party system nationalization is conceptualized in various ways. It is defined either in terms of territorial heterogeneity (more precisely – nationalization is treated as a growth of territorial homogeneity of party support), or in terms of inter-election incongruence (it is treated as a decrease of dissimilarities between various levels of party system organization). These ambiguities related to the measurement of nationalization have been already pointed out by Morgenstern and Pothoff (2005). These authors purposely did not include the term "nationalization" in their "components of election" model which consists of: district heterogeneity, volatility, and time-district effects.

Rose and Urwin (1975), as well as Caramani (2005), discussed various measures of the territorial heterogeneity of party support which generally describe "the dispersion of regional values [of party support] around the national mean" (Caramani, 2005: 299). In a similar vein, Jones and Mainwaring (2003: 139) analyze „the extent to which a party receives similar levels of electoral support throughout the country". In opposition to these formulations, Schakel and Swenden when discussing the nationalization of party systems, propose „to study party systems as multi-level party systems, i.e. by considering the performance of parties in regional elections and by relating this to their performance in general elections" (Schakel, Swenden, 2010: 2; Swenden and Maddens, 2009; Detterbeck and Hepburn 2009). Nationalization is treated in a similar vein by Kjaer and Elklit (2010) in their analyses of Danish local party system nationalization: "the degree of nationalization is higher in municipalities where local parties have won only a few council seats than where they have won a more substantial share of seats."

The first formulation refers to what we call in this article a territorial heterogeneity, while the second – what we consider inter-election incongruence. In the first case, the analyses take into account the differentiation of party support across territorial units, usually – electoral districts. In the second case – differentiation of party systems which stems from the existence of separate representative bodies at different levels of territorial organization of the country where a similar set of political parties compete for the seats (e.g. national parliament and regional assemblies).

Measures of inter-election incongruence are usually employed in the analyses focused on the multi-level or multi-layered (Deschouwer, 2003) construction of party systems or, alternatively, on the simultaneity of the elections to different representative bodies (inter-election split-ticket voting or inter-level ticket splitting; cf. Rallings, Trasher, 2003; Elklit, Kjaer, 2005). In his systematic study Schakel (2013) described three indices of dissimilarity between regional party systems and national party system: DIS1, which captures dissimilarity between the state-wide and the regional party system; DIS2, which captures the dissimilarity between the state-wide vote for the country as a whole and the statewide election result for a particular region; and DIS3, which captures the dissimilarity between the state-wide and regional vote for a particular region (Schakel, 2013: 633). Schakel clarifies that the variation of DIS2 can be ascribed to the specificity of regional electorates, the variation of DIS3 can be ascribed to the specificity of the regional elections, according to the second-order elections theories (Heath et al., 1999); finally, the variation of DIS1 accounts for the combination of both factors.

As we demonstrated, four main features of party systems enumerated at the beginning: disproportionality, volatility, territorial heterogeneity, and inter-election incongruence can be conceptualized as the dissimilarity of two vectors (distributions). However, two important issues still need to be clarified: (1) how these vectors should be specified, subject to the research problems; (2) which statistical indicators of similarity should be used, i.e. which indicators perform best as measures of different features of party systems. We will analyze these issues in the following paragraphs.

Alternative specifications of vectors

Disproportionality, volatility, and inter-election incongruence usually refer to the nation-wide party system; territorial heterogeneity is usually treated as a feature of separate political parties. However, a closer study on how the compared vectors are specified reveals numerous possibilities of dissimilarity formulation. In table 1 we present 14 pairs of vectors, the dissimilarity of which could have a meaningful interpretation for students of party systems. Different pairs refer either to the whole party system, or the particular parties, or the electoral districts (territorial units). We also propose a set of example research questions – each question can be addressed with the use of a particular dissimilarity index. This list is probably not exhaustive. However, it demonstrates that the operationalization of the abstract party systems features as the dissimilarities of vectors could be very productive.

[table 1 about here]

For example, the traditional approach to volatility requires the comparison of vectors presenting parties' support in two subsequent elections. Usually, dissimilarity between two elections is calculated for the whole country, but obviously, if more precise data are available, it could be calculated separately for all electoral districts. Nonetheless, the temporal stability of electoral support could be assessed also for a particular party, competing in both elections – less precisely if we take into account only aggregated results (two one-element vectors), more precisely if we compare party performance across districts in two following elections.

We argue that the main features describing party systems, as well as some important features of particular parties or electoral districts describing party systems indirectly, can be operationalized as dissimilarities of two vectors (distributions of votes or seats). Therefore, the problem of proper measurement of these features is actually a problem of selection between different measures of dissimilarity proposed by the statistics. In the following part of the article we discuss the main postulates for dissimilarity measures in political research and, subsequently, we verify whether these postulates are fulfilled by the most important measures of dissimilarity.

Measures

Our comparison of dissimilarity measures takes into account only some of these analyzed by Taagepera and Grofman (2003). We limited our list to these measures which in their basic form, or after a transformation, satisfy the following postulates:

1) ***Completeness***: the measure makes use of all data in X and Y and no additional information should be needed except the values of two vectors being compared to compute the measure,
2) ***Uniformity***: the measure treats the data uniformly, i.e. without giving a special role to the largest value (x_1, y_1) or two largest values,
3) ***Variability within [0,1] range***: the measure does not take negative values, nor it excesses 1,
4) ***0 limit***: the measure takes 0 value if and only if X = Y, i.e. 0 value reflects the perfect concordance between vectors
5) ***Insensitivity to scale transformation***: the measure should be invariant to the vectors' scale transformation (multiplication by positive number); in fact, this is closely related to the (3) postulate, as if measure was insensitive to scale transformation, it always has upper and lower limits; this postulate implies also insensitivity to shifts from fractional (0-1) to percent shares (0-100); thus, we present all the measures in their fractional form.

Specifically, we do not take into consideration measures based on the χ^2 distribution. Such measures were used, for example, by Nagel (1984) and Mudambi (1997) in their analyses of disproportionality. In our opinion, the usage of the p-levels related to the χ^2 statistics requires the assumption that there is a certain probabilistic model specified for the analyzed elections; in fact, it implies the test of hypothesis that two vectors were in some way "sampled" from the same underlying distribution. While such a probabilistic model is not defined precisely, it is not clear how we should interpret the values of χ^2-based probability which obviously depend on the sum of vectors' elements and are, therefore, sensitive to the scale transformations.

Let us assume that there are two vectors compared; they describe the distribution of votes or seats in certain elections: X ($x_1, x_2, \ldots x_k$) and Y ($y_1, y_2, \ldots y_k$). The dissimilarity between these vectors can be measured by the following measures:

1) **Index of disproportionality** (Duncan's, Pedersen's, or Loosemore-Hanby's index):

$$D = \frac{1}{2}\sum_i \left|\frac{y_i}{\sum_i y_i} - \frac{x_i}{\sum_i x_i}\right|$$

2) **Gallagher's least squares index** (i.e. length of vector's difference in the Euclidean metrics):

$$Gh = \sqrt{\frac{1}{2}\sum_i \left(\frac{y_i}{\sum_i y_i} - \frac{x_i}{\sum_i x_i}\right)^2}$$

3) **Squared Gallagher's index** although has not been widely used in research on party systems, we take it into account, as it can be decomposed (while Gallagher's index not):

$$Gh^2 = \frac{1}{2}\sum_i \left(\frac{y_i}{\sum_i y_i} - \frac{x_i}{\sum_i x_i}\right)^2$$

4) **Modified Gallagher's index**, proposed by Koppel and Diskin (2009); in this case, values of the vectors X and Y are normalized by the sum of *squared* values of all their elements, instead of sum of values of all their elements:

$$Gh' = \sqrt{\frac{1}{2}\sum_i \left(\frac{y_i}{\sqrt{\sum_i y_i^2}} - \frac{x_i}{\sqrt{\sum_i x_i^2}}\right)^2}$$

5) **Cosine measure**, introduced to the field of political science by Koppel and Diskin (2009); which equals the cosinus of the angle between vectors X and Y in Euclidean metrics; as originally it is a measure of similarity, it must be subtracted from one; it could be also presented as squared modified Gallagher's measure:

$$1 - cos = 1 - \frac{\sum_i y_i x_i}{\sqrt{\sum_i y_i^2 \sum_i x_i^2}} = (Gh')^2$$

6) **Modified Gini's coefficient.** We take into consideration this measure in the form equivalent to this

proposed by van Puyenbroeck (2008) and by Bochsler (2010) as "Gini coefficient corrected for unequal size of units". However, in certain articles, e.g. Penisi (1998), Jones and Mainwaring (2003) one could find a slightly different measures called "Gini's coefficient," as well. As van Puyenbroek indicates, these adaptations depart far from the ideas originally standing behind the Gini coefficient and they are of different formal properties (Puyenbroeck, 2008: 505-506)

$$Gini = 1 - 2\sum_i \frac{x_i}{\sum_j x_j}\left(\frac{\sum_{k=1}^{i} y_k - \frac{y_i}{2}}{\sum_j y_j}\right)$$

$$\text{if } X \text{ and } Y \text{ are sorted that } \forall_{l,m}: l > m \Rightarrow \frac{y_l}{x_l} \geq \frac{y_m}{x_m};$$

7) **Kullback-Leibler divergence** (relative entropy) is a measure derived from information theory (Kullback, Leibler, 1951) and can be interpreted as the amount of information lost when distribution Y is used to approximate X:

$$\mathrm{d_{KL}} = \sum_\mathrm{i}\left\{\frac{\mathrm{x_i}}{\sum_\mathrm{i}\mathrm{x_i}}\left[\log\left(\frac{\mathrm{x_i}}{\sum_\mathrm{i}\mathrm{x_i}}\right) - \log\left(\frac{\mathrm{y_i}}{\sum_\mathrm{i}\mathrm{y_i}}\right)\right]\right\} = \frac{1}{\sum_\mathrm{i}\mathrm{x_i}}\sum_\mathrm{i}\left(\mathrm{x_i}\log\frac{\mathrm{x_i}\sum_\mathrm{i}\mathrm{y_i}}{\mathrm{y_i}\sum_\mathrm{i}\mathrm{x_i}}\right) \text{ if } \forall_\mathrm{i}: \mathrm{x_i}, \mathrm{y_i} > 0$$

Kullback-Leibler divergence has no upper limit, but it can be normalized to the [0, 1] range when divided by $\log\sum_\mathrm{i}\mathrm{y_i}$. In practice, serious limitation for the use of this measure is the requirement that both vectors X and Y do not contain elements that have zero value; this drawback will be discussed in the following paragraphs.

8) **Weighted variance of ratios** has not been used explicitly as a measure of dissimilarity in political research. However, it could be related to the studies of territorial heterogeneity which analyze the unweighted variance of party's vote shares across districts (Caramani, 2005). Weighted variance is somehow analogous to "Gini coefficient corrected for unequal size of units" proposed by Bochsler (2010). Unlike ordinary (unweighted) variance, weighted variance of ratios could be viewed as a measure of dissimilarity of two vectors; for example, in the case of territorial heterogeneity the compared vectors are: the vector of votes casted for a particular party and the vector of all votes casted in each territorial unit.

$$V = Var_{w=X}\left(\frac{Y}{X}\right) = \sum_i \frac{x_i}{\sum_j x_j}\frac{y_i^2}{x_i^2} - \left[\sum_i \frac{x_i}{\sum_j x_j}\frac{y_i}{x_i}\right]^2 = \frac{1}{\sum_j x_j}\left[\sum_i \frac{y_i^2}{x_i} - \frac{(\sum_i y_i)^2}{\sum_j x_j}\right] \text{if } \forall_i: x_i > 0$$

In this form, the weighted variance of ratios does not have the upper limit, but it can be normalized to the [0, 1) range when divided by $\sum_i y_i$.

9) **Weighted standard deviation of ratios**, which is a transformation of weighted variance of ratios; it is expressed in the same unit as ratio Y/X, what facilitates its interpretation:

$$SD = \sqrt{V}$$

It can be also normalized to the range (0,1) analogically to the variance normalization.

Differences between measures: defining dimensions of comparison

In order to compare the above-mentioned measures of dissimilarity, let us formulate relevant postulates which could serve as dimensions on which various measures could perform differently. The most complex list of postulates for measures of dissimilarity, along with their justifications, was already presented by Taagepera and Grofman (2003). Some of them were already mentioned, as they served us as criteria of selection and, thus, they are fulfilled by all of the considered measures. For two vectors of equal length X, Y being compared the postulates are:

1) ***Symmetry x-y***: gives the same result for X~Y and Y~X comparisons; it should not matter which vector is placed first,
2) ***1 limit***: the measure takes 1 value if and only if $y_i=0$ for all $x_i>0$ and $x_i=0$ for all $y_i>0$ (X and Y are orthogonal), i.e. 1 value reflects the perfect dissimilarity between vectors; it is worth noticing that this postulate is more strict that the postulate of (0-1) range variability, as it refers to the condition of vectors' orthogonality.
3) ***Dalton's principle of transfers***
   a. ***ratio transfers***:

      If we set one of the two compared vectors as a reference – say X – we may define the relation of being *poorer* between the two elements of Y as: *i* is *poorer* than *j* if and only if $y_i/x_i<y_j/x_j$. Strong Dalton's principle of transfers demands that if Y' is such a transformation of Y that some value had been *transferred* from *richer* to *poorer* (formally: for some *i* being *poorer* than *j*: $y_i/x_i < y_i'/x_i < y_j/x_j$ and $y_i+y_j=y_i'+y_j'$), then dissimilarity between X and Y' is less than between X and Y. A weak version of the principle demands only that dissimilarity between X and Y' be not greater than between X and Y, but not necessarily less.

b. ***difference transfers*:**

Taagepera and Grofman (2003) proposed the modification of Dalton's principle of transfers to the form in which instead of ratios the differences are taken into account, while defining the relation of *being poorer*: *i* is *poorer* than *j* if and only if y_i-x_i<y_j-x_j.

It should be noted that a measure can meet Dalton's principle of transfers either in ratio form or in difference form, but not both simultaneously.

Another important postulate was introduced to the list by Koppel and Diskin (2009):

4) ***Optimality of equality***: if for some i, j: x_i=x_j and y_i=y_j, then if for some vector Y' $y_i+y_j=y_i'+y_j'$ and $y_i' \neq y_j'$, dissimilarity between X and Y' is greater than between X and Y.

The use of decomposed dissimilarity indices in the recent research on party systems motivated us to formulate a new postulate of the measure decomposability in a one of the two possible methods.

5) ***Decomposability***: assume that there are *k* groups and in each group there are vectors: X_j, Y_j of length l_j. If F(X, Y) is a measure of dissimilarity, the postulate of decomposition requests that if X and Y are vectors formed by concatenating X_j, Y_j for all j∈{1, ..., k} then F(X, Y) can be expressed solely as a function of aggregated characteristics of those groups (particularly F(X_j, Y_j) and perhaps other). We propose two forms of decomposition:

a. ***horizontal (sum-type)***: total value of measure can be expressed as a sum of values across groups:

$$F(X,Y) = \sum_j F(X_j, Y_j)$$

b. ***vertical (variance-type)***: total value of measure can be expressed as a sum of weighted mean of measure across groups (within group level) and value of measure computed on group-aggregated sums of X and Y (between group level):

$$F(X,Y) = \left(\frac{1}{\sum_{j=1}^{k} S_{Xj}}\right) \sum_{j=1}^{k} \left[S_{Xj} F\left(X_j, Y_j\right)\right] + F(S_X, S_Y)$$

where S_X and S_Y are vectors of sums of x_{ji} and y_{ji} among subsequent groups:

$$S_X = \left(\sum_{i=1}^{l_1} x_{1i}, \sum_{i=1}^{l_2} x_{2i}, \dots, \sum_{i=1}^{l_k} x_{ki}\right)$$

$$S_Y = \left(\sum_{i=1}^{l_1} y_{1i}, \sum_{i=1}^{l_2} y_{2i}, \dots, \sum_{i=1}^{l_k} y_{ki}\right)$$

Of course, these two forms of decomposition are mutually exclusive. It should be also noticed that vertical (variance-type) decomposition is contradictory to the postulate of symmetry, as vector X is treated in a different way than vector Y (vector X can be described as "weighting" or "reference") and the postulate of variability within [0,1] range, as measures which can be decomposed vertically do not have the upper limit. Although typically a certain form of normalization based on the sum of Y can be applied to compute the measure with a given upper limit, such normalized measure would not satisfy the postulate of decomposition.

The first method of decomposition (sum-type) in practice means that the value of the indicator could be presented as a sum of the values computed for the vector's fragments, distinguished on the basis of a certain criterion. For example, the horizontal decomposability allows to divide the net electoral volatility index into two parts: one which is a result of the changing electoral support for "stable" parties, i.e. participating both in t and (t-1) elections, and the other which is a result of entries to and exists from the party system, e.g. party splits or the establishment of "genuinely new" parties (Sikk, 2005). This is exactly the procedure used by Birch (2003) or, more recently, by Powell and Tucker (2013), who attempted to present separate models explaining two distinct types of volatility in new European democracies (Birch calls them "type I" and "type II", while Powell and Tucker "type A" and "type B").

Analogically, the horizontal decomposition could be also used to describe the components of the inter-election incongruence. The measure describing the dissimilarity between regional and national elections could be divided into two components: one related to the specificity of the "demand-side" (i.e. preferences of the regional electorates) and the other related to the specificity of the "supply-side" (differences in the "electoral menus" presented to the voters, stemming from the existence of regional parties or the absence of the nation-wide parties in certain regions).

In order to demonstrate the significance of the vertical decomposition of dissimilarity measures, one should refer to the research on territorial heterogeneity where the most popular methodological approach is based on the (unweighted) variance analysis (Stokes, 1967; Katz, 1973; Kawato, 1987; Morgenstern and Pothoff, 2005; Morgenstern, Swindle and Castagnola, 2009, Mustilo and Mustilo, 2012). As the unweighted variance cannot be treated as a measure of dissimilarity, it is difficult to present it within the frame adopted in this article. However, the existing research on territorial (district) heterogeneity serves well as an illustration of the idea of vertical decomposition, which could be also applied to the weighted variance of ratios.

The analyses of territorial heterogeneity demonstrate the variation of party's support across districts as a sum of two components: variation of mean support between regions and mean variation of support within regions. If the first component dominates, the party is considered as "regional" or "with regionally biased support". On the other hand, if the first component is rather low, the party is considered as "nationalized" with equally distributed support across the country. It is worth noticing that the procedure of variance analysis allows to conduct simultaneously a decomposition taking into account several factors, e.g. time and volatility of party support (Morgenstern and Potthoff, 2005; Morgenstern, Swindle and Castagnola, 2009). Some authors worked on the elaboration of models by adding various variables characterizing territorial units (Mustilo and Mustilo, 2012).

Nonetheless, in the existing research employing the approach based on the analysis of variance, the problem of unequal size of territorial units was omitted. Bochsler (2010) indicates this as one of the main disadvantages of this approach. The author points out that the territorial heterogeneity of party's support should be described by the number, not share, of votes, what implies weighting of territorial units based on their size. It should be mentioned that the methods of analysis for weighted variance are elaborated in various fields, e.g. in survey data analysis. There are no obstacles to use them also for multi-factor analysis, analogically to the work of Morgenstern and Pothoff (2005).

The weighted variance of ratios, which we use as a measure of dissimilarity, could be treated as a specific transformation of unweighted variance, which additionally allows the equal treatment of voters instead of equal treatments of territorial units. Additionally, it could be applied in a wider context than the territorial heterogeneity. For example, vertical decomposition could be used in the research on disproportionality, specifically – to answer the question on how the observed deviations from equal representation stem from two various sources: the seat apportionment to the electoral districts (variance between districts) and the effects of the electoral law within the districts. It would allow, for example, to describe more precisely, the electoral systems of federal countries or other situations in which the electoral districts for certain reasons have considerably different size.

Results

In Table 2 we collect the results of our analyses. The columns refer to 9 measures being compared, while each row corresponds with one out of 12 postulates (dimensions of assessment) which were discussed above.

[table 2 about here]

As it was already mentioned, all measures fulfill the postulates of completeness, uniformity, and 0 limit (defined as dissimilarity between two identical vectors). Two following postulates – insensitivity to scale transformation and variability within [0, 1] range – can be fulfilled by all the measures, while Kullback-Leibler divergence, weighted variance of ratios and weighted standard deviation of ratios require normalization, defined as division by: $\log \sum_i y_i$, $\sum_i y_i$ or $\sqrt{\sum_i y_i}$, respectively. Obviously, the condition that vector Y, and in case of K-L divergence also vector X, cannot contain any 0 elements implies, that these measures, even after normalization, cannot attain value of 1. However, one should notice that the normalized measures can fulfill this postulate asymptotically. When the sum of Y's elements for which X's elements have non-zero values tends to 0, the value of normalized weighted variance of ratios and weighted standard deviation of ratios tends to 1. Analogically, in case of K-L divergence; with the exception that also the sum of X's elements for which Y's elements have non-zero values should tend to 0.

Only Gallagher's coefficient and its squared form do not fulfill the "1 limit" postulate, i.e. they do not adopt value 1 in the case when the X and Y are orthogonal[2].

As far as the following postulates are concerned, it should be observed that the measures which satisfy the Dalton's principle of difference transfers are simultaneously symmetrical and insensitive for the transformations of scale in their basic form; this implies also the fact that they adopt values from [0,1] range. On the other hand, all of the measures which fulfill the Dalton's principle of ratio transfers are asymmetrical and, with the exception of Gini coefficient, they do not have upper limit in their basic form. The only measure which does not fulfill the optimality of equality is the dissimilarity index.

Before we proceed to the postulates of decomposability, we propose to distinguish three pairs of measures; in each pair the value of one measure is a squared value of the second. These pairs are: (1) *Gh* and Gh^2, (2) *Gh'* and *1-cos* (cosine measure of dissimilarity), (3) *V* and *SD*. In each pair, both measures fulfill exactly the same set of postulates with the exception for the postulates of decomposability. In each pair, only the second measure (squared) allows decomposition: in case of Gh^2 and *1-cos* it is horizontal decomposition, in case of *V* – vertical decomposition. Therefore, Gh^2, *1-cos* and *Var* should be treated as a primary measures in comparison with *Gh*, *Gh'* i *SD*. This observation has interesting consequences for the problem of identification of value referring to "halfway deviation from proportionality" or "half-perfect disproportionality", which was discussed by Taagepera and Grofman (2003: 672) and Koppel and Diskin (2009: 286). For Gh^2, *1-cos* and *Var* the „halfway deviation from proportionality" refers to the value of 0.5, while for the *Gh*, *Gh'* i *SD* it refers to the value of ~0.707.

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[2] It is worth noticing that the maximal possible value of Gallagher's index is equal to $\sqrt{\frac{1}{2}(1+\frac{1}{k-1})} = \sqrt{\frac{k}{2(k-1)}}$ - where *k* is the length of compared vectors. Therefore, for vectors which are longer than 2 elements its values are always lower than 1. For that reason, one should consider a normalization of this measure by dividing the raw *Gh* values by the maximal possible value which it can attain. It should be also stressed that the orthogonality of vectors X and Y is a necessary, but not sufficient condition of maximization of *Gh* values. Additionally, one of vectors should have only one zero element and the second vector – only one non-zero element.

One should admit that the measures which are square roots of decomposable measures have certain advantage – their values are expressed in fractions, i.e. in the units of original input data; thus they are easily interpretable. Generally, the utility of the above-mentioned measures should be assessed in pairs, as within the pair they share most important properties. In practice, the researcher should use one of them subject to his/her requirements.

Apart from the above-mentioned three pairs of indicators, KL divergence can be decomposed vertically, while index of dissimilarity – horizontally. Gini coefficient cannot be decomposed neither horizontally, nor vertically, what is a relatively important, but rarely noticed in the electoral studies, disadvantage of this measure[3].

Discussion

In the last section of this article, we will refer again to the four distinguished characteristics of party systems (disproportionality, volatility, territorial heterogeneity, and inter-election incongruence) and, on the basis of the analyses presented above, we will discuss which measures are the most appropriate for empirical studies in each of these four domains. In order to address this issue, we should consider the desired properties of measures of four discussed characteristics of party systems.

It seems that the methodological discussion about proper measurement of disproportionality is the most developed, thus the expectations towards disproportionality measures are relatively well discussed (Monroe, 1994; Grilli di Cortona et al., 1999; Balinski, Young, 2001; van Puyenbroeck, 2008). If we define disproportionality as the departure from the ideal situation in which each vote is equal (i.e. equal seat/vote

[3] Bochsler (2010) proposed a modification of Gini coefficient, which takes into account the unequal number of investigated units (e.g. electoral districts). However, this modification does not overcome the problem of decomposability of Gini coefficient; it is also based on quite strong assumptions concerning the territorial heterogeneity of support for particular parties.

ratio for each voter); thus, the dissimilarity measures serving as indicators of disproportionality should fulfill Dalton's principle of transfers in its original, i.e. ratio, form. This would ascertain that the indicator measures equality of „being represented" by all the voters. However, as van Puyenbroeck (2008) demonstrated, the measures fulfilling Dalton's principle of difference transfers (widely used Gallagher's coefficient, among them) refer to the concept of disproportionality as the departure from the ideal situation in which each party obtains the same share of seats as the share of votes it obtained in the elections. In this situation not only Dalton's principle of difference transfers is desired, but also the postulate of optimality of equality.

If we agree that in case of disproportionality measurement, the most important is Dalton's principle of ratio transfers, the most proper measures among these compared in our article are: Gini coefficient, K-L divergence, weighted variance of ratios and weighted standard deviation of ratios. One should admit that K-L divergence would not perform well as the measure of disproportionality, as it is common that some of the parties contesting elections obtain non-zero numbers of votes, but do not win any seats in the chamber. The choice between Gini coefficient and weighted variance of ratios (or weighted SD of ratios) could be regarded as a choice between the measure automatically normalized to [0,1] range, but non-decomposable and the measure decomposable but requiring additional normalization. As the decomposition of disproportionality measures is still not discussed in the political research, one should expect growing interest in Gini coefficient.

The measures which fulfill Dalton's principle of difference transfers and, simultaneously optimality of equality postulate, are Gh-Gh^2 and Gh'-(1-cos). Among them, the latter pair seems to have more intuitive condition of value maximization (i.e. orthogonality of vectors). Dunleavy and Margetts (1999) discuss, using the example of dissimilarity index, that the postulate of maximization of measure's values in case of vectors' orthogonality is irrelevant in the research on disproportionality. The orthogonality of seats' and votes' vectors would signify that the seats were assigned only for these parties which collected precisely zero votes; obviously, it could not happen under any electoral system. Nonetheless, Borisyuk et al. (2004) demonstrate that the modifications of dissimilarity index proposed by Dunleavy and Margetts are not a satisfying solution. It should be admitted that the criterion of maximization of measure's value in case of vectors' orthogonality is simple and intuitive, well rooted in the formal properties of the measures; even if in the case of disproportionality it refers to an impossible situation, any other solution seems to be more arbitrary and disputable.

In case of the studies on electoral volatility, the practical dominance of Pedersen index is unquestionable; thus, there is actually no discussion on the formal properties of alternative measures. Obviously, the concept of electoral volatility focuses on the party system. It leads to the selection of measures which „equally treat" each party, i.e. fulfill Dalton's principle of difference transfers. On the other hand, horizontal decomposability would allow to conduct analyses of the volatility components (Powell, Tucker, 2013), what is particularly important in case of dynamically changing party systems.

It should be noticed that the modification of Gallagher's coefficient, proposed by Koppel and Diskin (2009) should be preferred above the original version, as it additionally fulfills the postulate of value maximization in case of vectors' orthogonality. The pair *Gh'* and *1-cos* fulfills all the postulates which are fulfilled by dissimilarity index, and additionally the postulate of optimality of equality. It could lead to the suggestion that in the research on electoral volatility, *1-cos* or its square root (*Gh'*) should be preferred. However, the dominance of simple dissimilarity index would be probably unquestioned as the advantage of *1-cos* and *Gh'* refers to one criterion, which is not crucial in the research on electoral volatility.

In case of the research on territorial heterogeneity, it could be stressed that a proper approach should focus on "demand side", thus by treating equally each voter, regardless the size of territorial units (district) where the elections were held. As we already noticed in the case of disproportionality, it refers to Dalton's principle of ratio transfers. In case of territorial heterogeneity, the postulate of vertical decomposability seems to be of large importance, as the analyses usually compare the variation between territorial units with the variation within territorial units. In practice, the combination of these criteria lead to the elimination of all measures except from weighted variance of ratios and K-L divergence; however, the utility of K-L divergence is very limited, as we already demonstrated in the case of disproportionality – there are certain parties which in certain districts did not collect any votes as they did not cast their candidates. Therefore, it is worth promoting the usage of weighted variance of ratios what seems to be more promising solution than numerous attempts to modify and adapt Gini coefficient.

In case of inter-election incongruence, contrary to the analysis of territorial heterogeneity, the focus is put mainly on the "supply side", i.e. differences in parties' performance in various elections (including differences which stem from the participation only in one type of elections, e.g. only local/municipal). Analogically as in case of volatility, it leads to the preference for measures fulfilling Dalton's principle of difference transfers and measures which are horizontally decomposable. As a result, *1-cos* and its square root (*Gh'*) seems to be the most proper measure. However, similarly as in case of electoral volatility, the use of dissimilarity index is also justified due to its simple formula and more intuitive interpretation.

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Table 1. Conceptual construction of indices presenting features of parties, districts/regions and party systems – review

Feature of party system	Unit of reference	Two distributions compared	Questions / Dimensions of comparisons
Territorial heterogeneity	*district*	$v_p^{td} \sim v_p^t$	To what extent is the district specific?
	party	$v_d^{tp} \sim v_d^t$	To what extent is the party local/regional?
	a pair of parties	$v_d^{t(p1)} \sim v_d^{t(p2)}$	To what extent are the spatial distributions of support for two parties different?
	a pair of districts	$v_p^{t(d1)} \sim v_p^{t(d2)}$	To what extent are two districts different in terms of structure of party preferences?
Inter-election incongruence	*subsystem in relation to a whole system*	$v_p^{(e1)} \sim v_p^{(e2)r}$	To what extent is the subsystem (regional party system) dissimilar to the national party system?
	subsystem	$v_p^{(e1)r} \sim v_p^{(e2)r}$	To what extent is the structure of party preferences in a specific region different between regional and national elections?
	party	$v_r^{(e1)p} \sim v_r^{(e2)p}$	To what extent does the spatial distribution of party support differ between two elections of different type?
Disproportionality	*whole system by parties*	$s_p^t \sim v_p^t$	To what extent is the exactly proportional representation disturbed? What is the final effect of electoral law?
	whole system by districts	$s_d^t \sim v_d^t$	How high is the variation of the representation norm across various districts?
	district	$s_p^{td} \sim v_p^{td}$	To what extent is the exactly proportional representation disturbed in a certain district?
	party	$s_d^{tp} \sim v_d^{tp}$	To what extent does the distribution of seats refer to the distribution of votes for a certain party?
Volatility	*whole system*	$v_p^{(t+1)} \sim v_p^t$	How strongly did party system change?
	district	$v_p^{(t+1)d} \sim v_p^{td}$	How strongly did the local structure of political preferences change?
	party	$v_d^{(t+1)p} \sim v_d^{tp}$	How strongly did the spatial distribution of support for a certain party change?

Note: *v – stands for number of votes, s – number of seats. Indices describe: t – time of elections of the same type, p – parties, d – electoral districts (sometimes other territorial units, i.e. municipalities), r – regions (or other territorial units in which elections of a representative body take place), e1, e2 – elections of two different representative bodies, taking place simultaneously or in a possibly short time-span. Lower "counting" indices represent variables which are used to pair vectors, while upper indices represent variables specifying vectors which are compared.*

Table 2. A comparison of nine dissimilarity measures

Postulates	Measures								
	D	*Gh*	*Gh²*	*Gh'*	*1-cos*	*Gini*	d_{KL}	*SD*	*V*
Completeness	+	+	+	+	+	+	+	+	+
Uniformity	+	+	+	+	+	+	+	+	+
0 limit: F(X,X)=0	+	+	+	+	+	+	+	+	+
Insensitivity to scale transformation	+	+	+	+	+	+	*	*	*
Variablity within [0,1] range	+	***	***	+	+	+	**	**	**
Symmetry	+	+	+	+	+	-	-	-	-
1 limit: X⊥Y⇔F(X,Y)=1	+	-	-	+	+	+	**	**	**
Dalton's Principle of Transfers – ratios	-	-	-	-	-	+	+	+	+
Dalton's Principle of Transfers – differences	+	+	+	+	+	-	-	-	-
Optimality of equality	-	+	+	+	+	+	+	+	+
Decomposability – horizontal (sum-type)	+	-	+	-	+	-	-	-	-
Decomposability – vertical (variance-type)	-	-	-	-	-	-	+	-	+
Possible usage:									
Spatial heterogeneity	-	-		-		-	-	++	
Inter-election incongruence	+	+		++		-	-	-	
Disproportionality	-	+		++		+	-	++	
Volatility	+	+		++		-	-	-	

* normalization possible

** asymptotically, after possible normalization

*** for vectors of length greater than 2 maximum value is lower than 1, but normalization into (0, 1) is possible